# Calculating the Distance to the Moon

## Objectives

- Students will use mathematical formulas to calculate the distance from the Earth to the moon
- Students will use Astronomical Units to determine the distance to the moon
- Students will use proportions to calculate the distance from Earth to the moon in a model Earth/moon system
- Students will build models to proportion to accurately depict size and distance of the Earth and moon

## Suggested Grade Level

5th-8th

## Subject Areas

Science
Math

## Timeline

Two to three class periods

## Standards

***Science***

Earth and Space Science

- Earth in the solar system

Science in Personal and Social Perspectives

- Natural hazards
- Science and technology in society

***Math***

Algebra

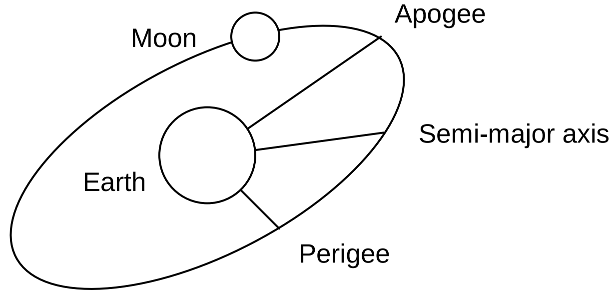
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships

Measurement

- Understand measurable attributes of objects and the units, systems, and processes of measurement
- Apply appropriate techniques, tools, and formulas to determine measurements

## Background

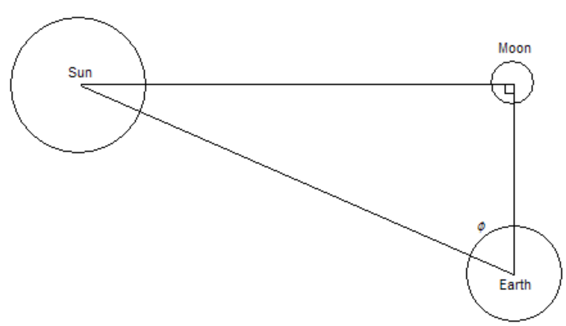
The moon is the Earth's only natural satellite. Because the moon orbits the Earth in an elliptical orbit the distance from Earth to the moon varies. The perigee of the moon or the closest point in its orbit is approximately 363,104 km (225,629 miles). The moon's apogee or farthest point in its orbit is approximately 405,696 km (252,095 miles). The semi-major axis of the moon's orbit is approximately 384,399 km (238,861 miles) distant from the Earth. The semi-major axis is the point half the distance between the perigee and apogee.



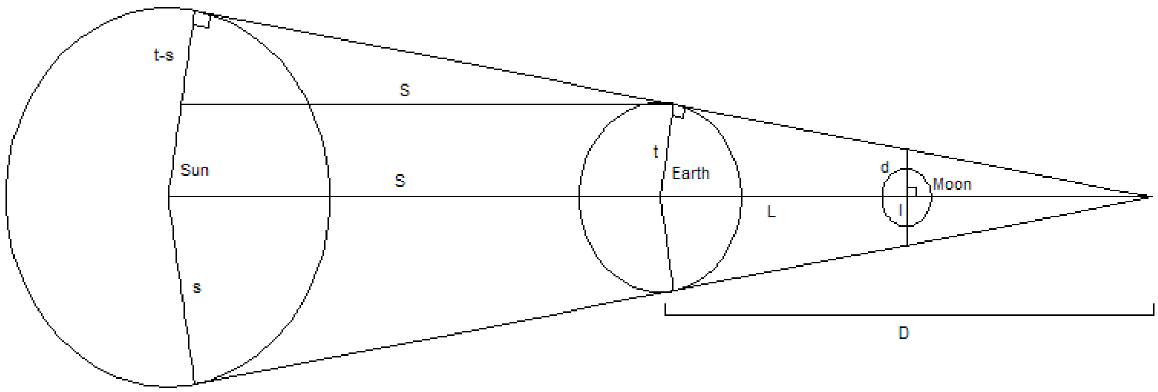

The Greek mathematician and astronomer, Aristarchus of Samos, who lived circa 310 BC – 230 BC, was the first to calculate the distances from Earth to the sun and moon. Around 270 BC he revealed his calculations about the size and distance of the sun and moon. His views, however, were not validated during his lifetime. His calculations put the sun not the Earth at the center of the solar system. This went against the thinking of the time of a geocentric universe where the Earth was the center of the universe. The "universe" at Aristarchus' time only consisted of the current solar system. Aristarchus stated that the universe was much bigger due to the parallax or lack of movement of stars in the sky. He concluded that these stars must be much farther away, therefore, the universe must be much bigger that was thought at the time.

Aristarchus calculated the distance to the sun and moon using basic trigonometry. He started with the premise that when the moon is half lit (quarter moon) it forms a right angle with the sun and Earth. He calculated that angle to be 87°. The actual angle is 89° 50'. Using this measurement, Aristarchus determined that the sun was 18 to 20 times greater distance from the Earth than the moon.

$$\frac{S}{L} = \frac{1}{\cos\varphi} = \sec\varphi.$$



Aristarchus then used a lunar eclipse to determine the distance from Earth to the moon. He measured the time it took for the moon to enter and exit the Earth's shadow during a lunar eclipse to determine the distance from Earth to the moon.




By similarity of the triangles, $\dfrac{D}{S} = \dfrac{t}{s-t}$ and $\dfrac{d}{t} = \dfrac{D-L}{D}$.

| Symbol | Meaning |
|---|---|
| s | radius of the sun |
| S | distance to the sun |
| l | radius of the moon |
| L | distance to the moon |
| t | radius of the Earth |
| D | distance to the vertex of Earth's shadow cone |
| n | d/l, a directly observable quantity during a lunar eclipse |

Aristarchus' logic was that if the sun is 20 times farther away from the Earth than the moon, then it must be 20 times larger. His logic is correct, but his calculations were incorrect. The sun is approximately 390 times farther away from the Earth than the moon. From his calculations he concluded that the moon was approximately 60 Earth radii distant.

In this lesson students will determine the distance of the moon from the Earth using Astronomical Units. Using that calculation as a reference, students will estimate the distance from Earth to the moon using a basketball and tennis ball as a model for the Earth and moon, respectively. Finally, using the model, students will calculate the correct distance that the tennis ball should be placed away from the basketball.

## Vocabulary

Astronomical unit, semi-major axis, apogee, perigee

## Materials

Basketball, tennis ball, modeling clay, 10 meters of butcher paper, masking tape, meter stick

## Lesson

1. Before the lesson lay the butcher paper on the floor or attach it to the wall so students can mark their estimates, using masking tape, for the distance from the Earth to the moon. You may also skip the use of butcher paper and have students make their estimates on the floor of the classroom or in a hallway. (Note: This may be used as an individual or group activity.)
2. Explain to the students that they will be calculating, in various ways, the distance from the Earth to the moon using ratios, proportions, algebraic formulas, and Astronomical Units. Then we will use that information to create a scale model of the Earth/moon system in the classroom.
3. Start by having the students calculate the distance from Earth to the moon using astronomical units to determine the distance in kilometers.
4. Provide the students with the following information:
   - Astronomical Unit is the unit of length of the semi-major axis of the Earth's orbit around the sun
   - 1 AU=150,000,000 km
   - The moon is 0.00256266 AU from Earth
5. With this information we can calculate the approximate distance from Earth to the moon.
6. One way to calculate this is to set up a proportion.

   $$\frac{\textbf{1AU}}{\textbf{150,000,000 km}} = \frac{\textbf{0.00256266 AU}}{x} = \textbf{384,399 km}$$

   **or**

   $$\frac{\textbf{(150,000,000 km x 0.00256266 AU)}}{\textbf{1AU}} = \textbf{384,399 km}$$

   Therefore, the distance from Earth to the moon is 384,399 km.
7. Now that we have calculated the distance from Earth to the moon we will use that information to determine how far away the moon will be from the Earth in our model.
8. Start by holding up the basketball. Explain to the students that the basketball will represent Earth.
9. The diameter of the Earth is 12,756 km.
10. The diameter of the basketball is 24 cm.
11. Place the basketball in a location where the students will be able to estimate and measure the distance to the moon.
12. Hold up the tennis ball. Explain to students that if the Earth were the size of a basketball then the moon would be relatively the size of the tennis ball.
13. The diameter of the moon is 3,476 km.
14. The diameter of the tennis ball is 6.9 cm.

15. Next, we must determine if these ratios are approximate enough for our purpose. We will do this by dividing the moon's actual diameter by the Earth's actual diameter.

**3,476 km/12,756 km = 0.27249 or 27.25%**

Therefore, the diameter of the moon is 27.25% that of Earth's diameter.

16. Now, we must determine if the diameter of the basketball is proportionate to that of the tennis ball. We will do this by dividing the tennis ball's diameter by the basketball's diameter.

**6.9 cm/24 cm = 0.2875 or 28.75%**

Therefore, the diameter of the tennis ball is 28.75% that of the basketball's diameter. (Note: This percentage is off slightly, but the amount is negligible for this demonstration. Also, the availability of a tennis ball is greater than other balls that are closer to the actual proportion.)

17. We have determined that the tennis ball and basketball are the approximate proportions as the Earth and moon.
18. Next, have students make an estimate of how far away the tennis ball should be placed to give an accurate representation of the actual Earth/moon distance. One idea is to have students put their initials on a piece of masking tape. Then, have the students place the tape on the floor or sheet of butcher paper to mark their estimate.
19. Once this has been completed explain to the students that we will calculate the distance from the Earth to the moon for our model system.
20. For this part of the exercise we will need to calculate how many kilometers 1 cm is equal to in our model.
21. To calculate this, we must divide 12,756 km (the diameter of the Earth) by 24 cm (the diameter of the basketball) to get another proportion. Explain to students that we can set up a proportion to calculate this.

$$\frac{24\text{ cm}}{1\text{ cm}} = \frac{12{,}756\text{ km}}{x} = 531.5\text{ km/cm} \quad \text{or} \quad 12{,}756\text{ km}/24\text{ cm} = 531.5\text{ km/cm}$$

Therefore, 1 cm in our model is equal to 531.5 km.

22. Now we will calculate the distance in our model.
23. The moon travels in an ellipse around the Earth. Therefore, its orbit has a perigee and apogee. Perigee is the point nearest the Earth in the orbit of the moon. The moon's perigee is 363,104 km. Apogee is the point farthest from the Earth in the orbit of the moon. The moon's apogee is 405,696 km.
24. For this model, however, we will use the semi-major axis of the moon's orbit. The semi-major axis is one-half the distance between the apogee and perigee in an elliptical orbit. The semi-major axis for the moon's orbit around the Earth is 384,399 km.

25. If we divide the semi-major axis of the moon by the number of kilometers per centimeter in our model we will get the distance from the Earth to the moon in our model.

$$\frac{384{,}399 \text{ km}}{531.5 \text{ km/cm}} = 723.2 \text{ cm} \quad \text{or} \quad 7.2 \text{ m}$$

Therefore, in our model, the distance from the Earth to the moon is 7.2 m.

26. On the butcher paper, mark the correct scaled distance so students can determine how accurate they were.

## Extensions

Another way to represent this model on a smaller scale is by using clay balls to represent the Earth and moon. You can use a clay ball that has a diameter of 10 cm to represent the Earth, and a 2.7 cm clay ball to represent the moon. For this model1 cm=1,275.6 km. Therefore, the distance from Earth to the moon in the model will equal 3 m.

## Evaluation/Assessment

Use the student worksheet to assess understanding.

## Resources

NASA. (1997). *Exploring the Moon: A Teacher's Guide with Activities for Earth and Space Sciences.* (NASA EG-1997-10-116-HQ).

Wikipedia: Aristarchus of Samos.
http://en.wikipedia.org/wiki/Aristarchus_of_Samos

Wikipedia: Aristarchus, On the Sizes and Distances (of the Sun and Moon).
http://en.wikipedia.org/wiki/Aristarchus_On_the_Sizes_and_Distances

Name ____________________

# Calculating the Distance to the Moon

1. Calculate the distance to the moon using Astronomical Units. Solve for *x*.

   1 AU = 150,000,000 km
   0.00256266 AU = distance from the moon to the Earth
   *x* = moon's distance from Earth in km
   Use a proportion to solve.

   $$\frac{\text{1AU}}{\text{Length of 1AU in km}} = \frac{\text{Moon's distance in AU}}{x}$$

   $$\frac{\text{1AU}}{\_\_\_\_\_\_\_\_\_\_} = \_\_\_\_\_\_\_\_\_\_$$

   *x* = ____________________________

2. For our model we want to use a basketball to represent the Earth and a tennis ball to represent the moon. We must first determine if the proportion of the Earth to the moon and the basketball to the tennis ball are comparable. We can do this by dividing the moon's diameter by the Earth's diameter.

   The diameter of the moon is 3,476 km.
   The diameter of the Earth is 12,756 km.

   ____________________ = ____________________ or __________%

   Now we can determine the proportion of the moon to the basketball.
   The diameter of the tennis ball is 6.9 cm.
   The diameter of the basketball is 24 cm.

   ____________________ = ____________________ or __________%

Is the basketball and tennis ball proportionate enough to one another as to the Earth and moon so we can use them in our demonstration?

Yes No

Explain ____________________________________________________

____________________________________________________________

3. The teacher has placed the basketball at a point in the room. Estimate the distance the tennis ball must be placed away from the Earth to make this model accurate. (Use metric values,)

   Estimate: ______________________________

4. We must now calculate the distance, in kilometers, that 1 cm will represent in our model. We can do this by setting up a proportion with the values that we already know.

   12,756 km = the diameter of the Earth
   24 cm = the diameter of the basketball
   *x* = the distance that 1 cm will equal in our model

   $$\frac{\text{Diameter of the basketball}}{1\text{ cm}} = \frac{\text{Diameter of Earth}}{x}$$

   $$\frac{\rule{4cm}{0.4pt}}{\phantom{x}} = \frac{\rule{4cm}{0.4pt}}{x}$$

   *x* = ___________________________km/cm

5. Next, we will determine how far the tennis ball will be placed from the basketball in our model. We can calculate this by dividing the actual distance to the moon found in problem #1 by the value we calculated in problem #4.

   $$\frac{\text{Distance to the moon in km}}{\text{Value from problem \#4}} = x$$

   ____________________ = *x*

   *x* = _________________________ cm or _________________________ m